itly imposed at the interface between media with different permittivities. It is found from Fig. 1 that the agreement is very good between the results obtained by the present formulation (solid lines) and those obtained by the formulation using the transverse magnetic field component (hollow circles) [2]. The spurious solutions do not appear in the entire region of a propagation diagram.

As a more complicated waveguiding configuration, we next consider the rectangular waveguide with a diamond-shaped insert studied by Csendes and Silvester [17], as illustrated in Fig. 2. In this waveguide, there are abrupt changes in the permittivity at the interface, the normal direction of which does not coincide with the direction of a coordinate axis. Fig. 2 shows the dispersion characteristics for the fundamental mode, where two planes of symmetry are assumed to be perfect magnetic conductors and one quarter of the cross section is divided into second-order triangular elements. In Fig. 2, the results of the $\boldsymbol{H}_t$ field formulation [2] with $N_E = 50$ and $N_p = 121$ and those of the modal approximation technique [17] are also presented. For $\epsilon_1 = 1.5$, the results of the present $\boldsymbol{E}_t$ field formulation with $N_E = 50$ and $N_p = 121$ agree well with those of the $\boldsymbol{H}_t$ counterpart. On the other hand, for a larger value of relative permittivity, $\epsilon_1 = 10$, the results of the $\boldsymbol{E}_t$ field formulation with $N_E = 50$ deviate from those of the $\boldsymbol{H}_t$ counterpart at higher frequencies. This deviation at higher frequencies reflects the singularity of the normal electric field component at the tips of wedges of the dielectric insert [18]–[20]. Such a singularity near the tip of a dielectric corner may cause electrical breakdown in high-power applications. It is evident from Fig. 2 that the $\boldsymbol{E}_t$ field finite element solutions can be improved by increasing the number of elements. Indeed, the results of the $\boldsymbol{E}_t$ field formulation with $N_E = 128$ and $N_p = 289$ are closer to those of the $\boldsymbol{H}_t$ field counterpart. No spurious solutions are involved in this case as well.

IV. Conclusions

We have formulated a vectorial finite element scheme for solving guided-wave problems using the transverse electric field component. Considering the duality between electric and magnetic field vectors in Maxwell's equations, we have used the same procedure as the transverse magnetic field counterpart except that additional conditions are enforced on the boundary between different dielectric materials. In this approach, the electric field components can be directly obtained as an eigenvector of a matrix eigenvalue problem. Furthermore, no spurious solutions are involved in the entire region of a propagation diagram, and the dimension of the final matrix equation is reduced to two thirds that of the penalty function method. We have confirmed the validity of the present formulation via applications to some canonical waveguide problems.

The approach described in this paper is also applicable to waveguides containing anisotropic media such as ferrites because the tensor permeability may vary from material to material. Furthermore, extension to waveguides containing lossy and/or active media is straightforward if one uses the procedure that has recently been proposed by the authors [21] for the magnetic field case.

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An Improved Algorithm for the Computer-Aided Design of Coupled Slab Lines

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***Abstract* — Improved analytical formulas for the computer-aided design of parallel coupled slab lines are presented. These formulas ensure a good agreement of calculations with accurate numerical results for a wide range of geometrical dimensions of the lines.**

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I. Introduction

The evaluation of the characteristic impedances of coupled slab lines has been a subject of investigation for many years [1]–[6]. As confirmed in [6], the numerical results recently published in [5] are useful in practice from the standpoint of accuracy. These results, however, cannot be directly used in computer subroutines for design. It is worth noting that the interpolating expressions derived in [5] are of little help, because they show relatively poor agreement with the accurate numerical results obtained by means of rigorous methods. Therefore in this paper new, more accurate analytical formulas for computer-aided design of coupled slab lines are presented. They make it possible to calculate the geometrical dimensions of the lines for given characteristic impedances Z_{0e} and Z_{0o}. The design of lines under consideration is equivalent to solving two nonlinear equations with the diameter of the rods and the width of the slot as unknown variables. For this purpose conventional numerical methods can be applied.

Simple expressions for calculating the first approximation of the solution being sought are given.

The advantage of the proposed design algorithm lies also in its simple mathematical form, enabling it to be easily implemented even on small personal computers.

II. Design Algorithm

A transverse section of the parallel coupled slab lines is shown in Fig. 1. If the values of the characteristic impedances Z_{0e} and Z_{0o} are given, then the design of these lines consists in calculating $x = d/h$ and $y = s/h$ from the following set of equations:

$$
\begin{aligned}
V_1(x,y) &= Z_{0e}(x,y) - Z_{0e} = 0\\
V_2(x,y) &= Z_{0o}(x,y) - Z_{0o} = 0
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
Z_{0e}(x,y) &= 59.952 \ln\left[0.523962/\left(f_1(x) f_2(x,y) f_3(x,y)\right)\right]\\
Z_{0o}(x,y) &= 59.952 \ln\left[0.523962 f_3(x,y)/\left(f_1(x) f_4(x,y)\right)\right]\\
f_1(x) &= x a(x)/b(x)\\
f_2(x,y) &= \begin{vmatrix} c(y) - x d(y) + e(x) g(y) & \text{for } y < 0.9\\ 1 + 0.004 \exp(0.9 - y) & \text{for } y \geqslant 0.9 \end{vmatrix}\\
f_3(x,y) &= \tanh\left[\pi(x+y)/2\right]\\
f_4(x,y) &= \begin{vmatrix} k(y) - x l(y) + m(x) n(y) & \text{for } y < 0.9\\ 1 & \text{for } y \geqslant 0.9 \end{vmatrix}\\
a(x) &= 1 + \exp(16x - 18.272)\\
b(x) &= \sqrt{5.905 - x^4}\\
c(y) &= -0.8107 y^3 + 1.3401 y^2 - 0.6929 y + 1.0892\\
&\quad + 0.014002/y - 0.000636/y^2\\
d(y) &= 0.11 - 0.83 y + 1.64 y^2 - y^3\\
e(x) &= -0.15 \exp(-13x)\\
g(y) &= 2.23 \exp(-7.01 y + 10.24 y^2 - 27.58 y^3)\\
k(y) &= 1 + 0.01(-0.0726 - 0.2145/y + 0.222573/y^2\\
&\quad - 0.012823/y^3)\\
l(y) &= 0.01(-0.26 + 0.6866/y + 0.0831/y^2\\
&\quad - 0.0076/y^3)\\
m(x) &= -0.1098 + 1.2138 x - 2.2535 x^2 + 1.1313 x^3\\
n(y) &= -0.019 - 0.016/y + 0.0362/y^2 - 0.00243/y^3.
\end{aligned}
\tag{2}
$$

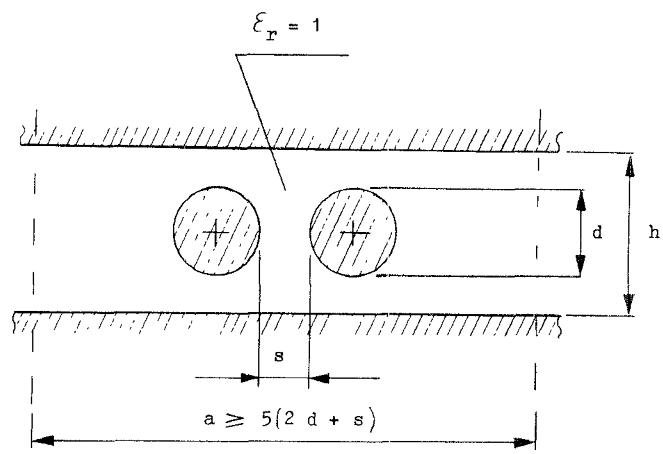

Fig. 1. Transverse section of the parallel coupled slab lines.

The algorithm for solving the set of equations (1) should be accurate, reliable, quickly convergent, and simple. For solving these equations a few different numerical methods have been tried. From the performed analysis it follows that the best results, in the sense of the above requirements, can be achieved by using the conventional Newton's method. According to [7], the $(n+1)$th approximation of the solution being sought is evaluated as follows:

$$
\begin{aligned}
x^{(n+1)} &= x^{(n)} - \frac{1}{J}\left(V_1 \frac{\partial V_2}{\partial y} - V_2 \frac{\partial V_1}{\partial y}\right)\\
y^{(n+1)} &= y^{(n)} + \frac{1}{J}\left(V_1 \frac{\partial V_2}{\partial x} - V_2 \frac{\partial V_1}{\partial x}\right)
\end{aligned}
\tag{3}
$$

where

$$
J = \begin{bmatrix} \dfrac{\partial V_1}{\partial x}, & \dfrac{\partial V_1}{\partial y}\\[2mm] \dfrac{\partial V_2}{\partial x}, & \dfrac{\partial V_2}{\partial y} \end{bmatrix} \neq 0. \tag{4}
$$

Here $(x^{(n)}, y^{(n)})$ is the nth approximation, $n = 0, 1, 2, \cdots$.

It should be noted that functions $V_1(x,y)$, $V_2(x,y)$ and their first partial derivatives (see formulas (3) and (4)) are calculated at the point $(x^{(n)}, y^{(n)})$. The initial approximation of the solution can be evaluated by

$$
\begin{aligned}
x^{(0)} &= 4/\pi \exp\left[-Z_0\Big/\left(59.952\sqrt{0.987 - 0.171k - 1.723k^2}\right)\right]\\
y^{(0)} &= 1/\pi \ln\left[(r+1)/(r-1)\right] - x^{(0)}
\end{aligned}
\tag{5}
$$

where

$$
\begin{aligned}
Z_0 &= \sqrt{Z_{0e} Z_{0o}}\\
k &= (Z_{0e} - Z_{0o})/(Z_{0e} + Z_{0o})\\
r &= \left[4\Big/\left(\pi x^{(0)}\right)\right]^{(0.001 + 1.117k - 0.683k^2)}
\end{aligned}
$$

It is worth pointing out that the above initial approximation, entered into a computer program, significantly reduces the computation time and makes this program more reliable and effective.

III. Computational Results

As a basis for evaluation of the presented design formulas the accurate numerical results published in [5] have been taken. These results are reported in terms of the variables $x = d/h$ and $y = s/h$ in Tables I and II.

TABLE I
NUMERICAL VALUES OF THE EVEN-MODE CHARACTERISTIC IMPEDANCE Z_{0e}, IN Ω

y \ x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.1	220.68	155.34	116.39	89.37	69.13	53.12	39.83	-
0.2	200.63	143.73	109.10	84.68	66.10	51.21	-	27.61
0.3	187.06	135.08	103.46	80.96	63.66	-	37.76	-
0.4	177.56	128.69	99.16	78.04	-	48.41	-	26.71
0.5	170.76	124.01	95.91	-	60.22	-	36.39	-
0.6	165.83	120.53	-	74.13	-	46.66	-	26.12
0.7	162.25	-	91.67	-	58.21	-	35.57	-
0.8	-	116.11	-	71.92	-	45.65	-	25.78
0.9	157.72	-	89.34	-	57.01	-	35.09	-
1.0	-	113.73	-	70.71	-	45.09	-	25.59
1.1	155.31	-	88.10	-	56.44	-	34.83	-
1.2	-	112.44	-	70.05	-	44.78	-	25.48
1.3	154.03	-	87.42	-	56.11	-	34.69	-
1.4	-	111.76	-	69.70	-	44.61	-	-
1.5	153.34	-	87.05	-	55.92	-	-	-
1.6	-	111.41	-	69.51	-	-	-	-
1.7	152.96	-	86.85	-	-	-	-	-
1.8	-	111.20	-	-	-	-	-	-
1.9	152.78	-	-	-	-	-	-	-
2.0	-	-	-	-	-	-	-	-

TABLE II
NUMERICAL VALUES OF THE ODD-MODE CHARACTERISTIC IMPEDANCE Z_{0o}, IN Ω

y \ x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.1	77.50	55.23	44.12	36.79	31.18	26.44	22.06	-
0.2	101.89	73.26	58.20	47.94	39.95	33.17	-	20.83
0.3	116.86	84.46	66.81	54.59	45.02	-	29.56	-
0.4	126.96	92.03	72.57	58.95	-	39.24	-	23.40
0.5	134.05	97.31	76.55	-	50.45	-	32.17	-
0.6	139.12	101.09	-	64.01	-	41.84	-	24.41
0.7	142.77	-	81.37	-	53.02	-	33.34	-
0.8	-	105.76	-	66.53	-	43.09	-	24.86
0.9	147.35	-	83.86	-	54.30	-	33.90	-
1.0	-	108.21	-	67.83	-	43.72	-	25.10
1.1	149.77	-	85.17	-	54.98	-	34.20	-
1.2	-	109.51	-	68.52	-	44.05	-	25.22
1.3	151.08	-	85.85	-	55.32	-	34.35	-
1.4	-	110.19	-	68.88	-	44.22	-	-
1.5	151.76	-	86.22	-	55.50	-	-	-
1.6	-	110.56	-	69.08	-	-	-	-
1.7	152.12	-	86.41	-	-	-	-	-
1.8	-	110.74	-	-	-	-	-	-
1.9	152.32	-	-	-	-	-	-	-
2.0	-	-	-	-	-	-	-	-

The formulas (1) and (2) together with the expressions (5) have been used in the computer program CSL. In this program the auxiliary function $U(x, y)$, defined as

$$U(x, y) = V_1^2(x, y) + V_2^2(x, y) \tag{6}$$

is also used. The search for the solution (x, y) terminates if $U(x, y) \leqslant Z_{0e}Z_{0o}/1\,000\,000$, which ensures relative accuracy of the approximation not worse than 0.1 percent for given impedances Z_{0e} and Z_{0o}.

In the program (see the Appendix) apart from computation statements, there are instructions for protection from different kinds of errors. For instance, calculations will not be made if $Z_{0e} \leqslant Z_{0o}$ and then the appropriate report is printed. Similarly, the Jacobian given by (4) should assume nonzero values during all iterations. Errors due to dividing by zero, although they are of slight probability, are eliminated by means of instructions written in lines 500, 510, and 520 of the program.

Some illustrative results, computed by means of CSL, are given in Table III. In the numerical experiment all values of impedances

TABLE III
COMPUTATIONAL RESULTS

k	Z_0, ohm	$x^{(0)}$	$y^{(0)}$	x	y
0.031623	50	0.5483	0.7882	0.5483	0.7882
0.056234	50	0.5464	0.6144	0.5460	0.6119
0.100000	50	0.5418	0.4439	0.5418	0.4439
0.177828	50	0.5291	0.2829	0.5286	0.2802
0.316227	50	0.4893	0.1460	0.4893	0.1460

Remark: The characteristic impedances Z_{0e} and Z_{0o} are related to the impedance Z_0 and the coupling coefficient k as follows:

$$Z_{0e} = Z_0\sqrt{\frac{1+k}{1-k}} \qquad Z_{0o} = Z_0\sqrt{\frac{1-k}{1+k}}.$$

Z_{0e} and Z_{0o} given in Tables I and II have been compared with the corresponding values computed with (2).

It has been found that formulas (2) ensure a very good agreement of calculations with the accurate numerical results for a wide range of geometrical dimensions. For $0.1 \leqslant x \leqslant 0.8$ and $0.1 \leqslant y$ the maximum percent deviation between the numerical results (see Table I) and the interpolated values is not greater than 0.27 percent for the impedance Z_{0e}. For comparison, the corresponding deviation obtained with the interpolating expressions derived in [5] is 1.44 percent. Similarly, for $0.1 \leqslant x \leqslant 0.8$ and $0.1 \leqslant y$ the maximum relative deviation evaluated for the impedance Z_{0o} (see Table II) is equal to 0.65 percent. In this case the corresponding deviation involved when using the interpolating expressions given in [5] is greater than 3.65 percent. The average arithmetical deviations computed with the proposed formulas for impedance Z_{0e} and Z_{0o} are equal to 0.13 percent and 0.15 percent. The comparative values of the average deviations calculated using the interpolating expressions [5] are equal to 0.42 percent and 0.70 percent, respectively.

IV. CONCLUSIONS

The design formulas presented in this paper make it possible to access the geometrical dimensions of parallel coupled slab lines provided their characteristic impedances Z_{0e} and Z_{0o} are known. It has been found from the numerical analysis that the presented formulas ensure very good agreement of calculations with the corresponding accurate values given in [5]. For $0.1 \leqslant x \leqslant 0.8$ and $0.1 \leqslant y$ the maximum percent deviations between the accurate (see Tables I and II) and interpolated values of impedances Z_{0e} and Z_{0o} are equal to 0.27 percent and 0.65 percent, respectively. The design of lines is transformed into the problem of solving two nonlinear algebraic equations with the diameter of the rods and the width of the slot as unknown variables. Simple expressions for calculating the first approximation of the solution being sought are given.

The advantage of the proposed design algorithm lies also in its simple mathematical form, which can be easily implemented even on small personal computers.

The solution procedure is also presented, which allows the practical realization of accurate, reliable, quickly convergent, and relatively simple computer programs. As an example of such a program the CSL (in BASIC) is given in the Appendix.

APPENDIX

```
10 REM CSL
20 PRINT "COUPLED SLAB LINES"
30 PRINT
40 PRINT "by S.Rosloniec, Warsaw 1988"
50 PRINT
60 PRINT "Data:"
70 PRINT
80 PRINT "Zoe=";: INPUT zoe;: PRINT zoe,"ohm"
90 PRINT "Zoo=";: INPUT zoo;: PRINT zoo,"ohm"
100 PRINT " er=1"
110 PRINT " ur=1"
120 PRINT
130 IF zoe<zoo THEN PRINT "Error: Zoe<Zoo": GO TO 0050
140 IF zoe=zoo THEN PRINT "The lines are not coupled.": STOP
150 PRINT "Please wait !"
160 PRINT
170 LET z=SQR (zoe*zoo)
180 LET k=(zoe-zoo)/(zoe+zoo)
190 LET xo=4/PI*EXP (-z/(59.952*SQR (.987-.171*k-1.723*k*k)))
200 LET r=(4/(PI*xo))↑(.001+1.117*k-.683*k*k)
210 LET yo=ABS (1/PI*LN ((r+1)/(r-1))-xo)
220 LET x=xo
230 LET y=yo
240 GO SUB 0300
250 PRINT "Results:"
260 PRINT
270 PRINT "d/h=";x
280 PRINT "s/h=";y
290 STOP
300 REM Procedure CSL
310 GO SUB 0590
320 LET uo=u
330 LET vo=v
340 IF (uo*uo+vo*vo)<z*z/1000000 THEN RETURN
350 LET dh=.0001
360 LET x=x+dh
370 GO SUB 0590
380 LET u1=u
390 LET v1=v
400 LET x=x-dh
410 LET y=y+dh
420 GO SUB 0590
430 LET u2=u
440 LET v2=v
450 LET d1=(u1-uo)/dh
460 LET d2=(u2-uo)/dh
470 LET d3=(v1-vo)/dh
480 LET d4=(v2-vo)/dh
490 LET det=d1*d4-d2*d3
500 IF ABS (det)>1e-9 THEN GO TO 0530
510 LET x=1.01*x
520 GO TO 0310
530 LET x=ABS (x-(uo*d4-vo*d2)/det)
540 LET y=ABS (y+(uo*d3-vo*d1)/det)
550 IF y<.09 THEN PRINT "Remark: s/h<.1": STOP
560 IF x<.09 THEN PRINT "Remark: d/h<.1": STOP
570 IF x>.81 THEN PRINT "Remark: d/h>.8": STOP
580 GO TO 0310
590 REM Subroutine 0590
600 LET a=1+EXP (16*x-18.272)
610 LET b=SQR (5.905-x*x*x*x)
620 LET h=EXP (PI*(x+y)/2)
630 LET f1=x*a/b
640 LET f3=(h*h-1)/(h*h+1)
650 IF y>=.9 THEN LET f2=1+.004*EXP (.9-y): LET f4=1: GO TO 0760
660 LET c=-.8107*y↑3+1.3401*y↑2-.6929*y+1.0892+.014002/y-.000636/y↑2
670 LET d=.11-.83*y+1.64*y↑2-y↑3
680 LET e=-.15*EXP (-13*x)
690 LET g=2.23*EXP (-7.01*y+10.24*y↑2-27.58*y↑3)
700 LET k=1+.01*(-.0726-.2145/y+.222573/y↑2-.012823/y↑3)
710 LET l=.01*(-.26+.6866/y+.0831/y↑2-.0076/y↑3)
720 LET m=-.1098+1.2138*x-2.2535*x↑2+1.1313*x↑3
730 LET n=-.019-.016/y+.0362/y↑2-.00243/y↑3
740 LET f2=c-x*d+e*g
750 LET f4=k-x*l+m*n
760 LET ze=59.952*LN ABS (.523962/(f1*f2*f3))
770 LET zo=59.952*LN ABS (.523962*f3/(f1*f4))
780 LET u=ze-zoe
790 LET v=zo-zoo
800 RETURN
```

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